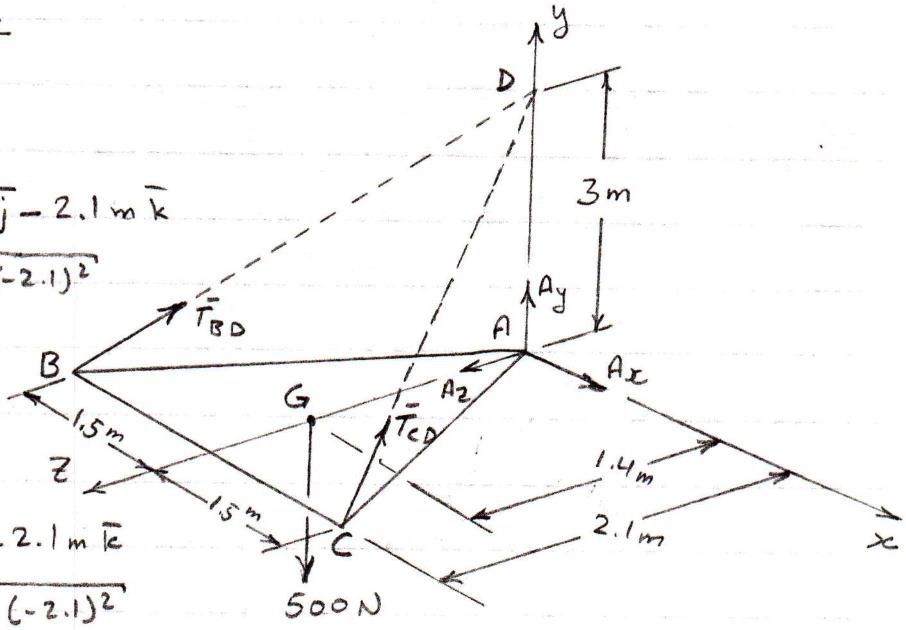
GNG 1105 A & E
ENGINEERING MECHANICS
FINAL EXAM
SOLUTIONS

Dec. 12, 2014

1. a) FBD - plate ABC

See diagram.



b) $\overline{BD} = 1.5m\bar{i} + 3m\bar{j} - 2.1m\bar{k}$

$BD = \sqrt{(1.5)^2 + (3)^2 + (-2.1)^2}$
$= 3.96m$

$\overline{CD} = -1.5m\bar{i} + 3m\bar{j} - 2.1m\bar{k}$

$CD = \sqrt{(-1.5)^2 + (3)^2 + (-2.1)^2}$
$= 3.96m.$

$\bar{T}_{BD} = T_{BD}\bar{\lambda}_{BD} = T_{BD}\dfrac{\overline{BD}}{BD}$
$= \dfrac{T_{BD}}{3.96}(1.5\bar{i} + 3\bar{j} - 2.1\bar{k})$ —— ANS

$\bar{T}_{CD} = T_{CD}\bar{\lambda}_{CD} = T_{CD}\dfrac{\overline{CD}}{CD}$
$= \dfrac{T_{CD}}{3.96}(-1.5\bar{i} + 3\bar{j} - 2.1\bar{k})$ —— ANS.

$\bar{W} = -500N\bar{j}$ —— ANS.

c) $BA = CA = \sqrt{(2.1)^2 + (1.5)^2} = \sqrt{6.66} = 2.58m.$

$\Sigma\bar{M}_A = \bar{r}_{B/A}\bar{T}_{BD} + \bar{r}_{C/A}\bar{T}_{CD} + \bar{r}_{G/A}\bar{W} = 0$

where $\bar{r}_{B/A} = -1.5\bar{i} + 2.1\bar{k}$, $\bar{r}_{C/A} = 1.5\bar{i} + 2.1\bar{k}$; $\bar{r}_{G/A} = 1.4\bar{k}$

(Cont'd on next page)

1. c) Cont'd.

$$\Sigma \bar{M}_A = (-1.5\bar{i} + 2.1\bar{k}) \times \frac{T_{BD}}{3.96}(1.5\bar{i} + 3.0\bar{j} - 2.1\bar{k})$$

$$+ (1.5\bar{i} + 2.1\bar{k}) \times \frac{T_{CD}}{3.96}(-1.5\bar{i} + 3.0\bar{j} - 2.1\bar{k}) + 1.4\bar{k} \times (-500N)\bar{j} = 0$$

$$\Sigma \bar{M}_A = -1.14 T_{BD}\bar{k} - \cancel{0.80 T_{BD}\bar{j}} + \cancel{0.80 T_{BD}\bar{j}} - 1.59 T_{BD}\bar{i}$$

$$+ 1.14 T_{CD}\bar{k} + \cancel{0.80 T_{CD}\bar{j}} - \cancel{0.80 T_{CD}\bar{j}} - 1.59 T_{CD}\bar{i} \quad \bar{k}\ \bar{j}\ \bar{i}$$

$$+ 700\bar{i} = 0$$

$$\therefore \Sigma \bar{M}_A = -1.14 T_{BD}\bar{k} - 1.59 T_{BD}\bar{i} + 1.14 T_{CD}\bar{k} - 1.59 T_{CD}\bar{i} + 700\bar{i} = 0$$

Equate Coefficient of $\bar{i}$ & $\bar{k}$ to zero:

$\bar{i}$: $-1.59\, T_{BD} - 1.59\, T_{CD} + 700 = 0$ —— (1)

$\bar{k}$: $-1.14\, T_{BD} + 1.14\, T_{CD} = 0$ —— (2)

$\therefore$ From eq. (2): $T_{BD} = T_{CD}$ which is obvious due to Symmetry

$\therefore$ eq (1): $-1.59 T_{BD} - 1.59 T_{BD} + 700 = 0$

$3.18\, T_{BD} = 700$

Hence, $T_{BD} = T_{CD} = \dfrac{700}{3.18} = \underline{\underline{220.13N}} = \underline{\underline{220N}}$ ANS.

$\Sigma F_x = 0$

$A_x + \frac{T_{BD}}{3.96} \times 1.5 - \frac{T_{CD}}{3.96} \times 1.5 = 0$

But, $T_{BD} = T_{CD}$,

$\therefore \underline{\underline{A_x = 0}}$ ANS.

$\Sigma F_y = 0 = A_y + \frac{T_{BD}}{3.96} \times 3.0 + \frac{T_{CD}}{3.96} \times 3.0 - 500N$

$\therefore A_y + \frac{220.13}{3.96} \times 3.0 + \frac{220.13}{3.96} \times 3.0 - 500N = 0$

Hence, $\underline{\underline{A_y}} = -166.77 - 166.77 + 500N = \underline{\underline{166.46N}}$ ANS.

$\Sigma F_z = 0 = A_z - \frac{T_{BD}}{3.96} \times 2.1 - \frac{T_{CD}}{3.96} \times 2.1$

$\therefore A_z - \frac{220.13}{3.96} \times 2.1 - \frac{220.13}{3.96} \times 2.1 = 0$

Hence, $\underline{\underline{A_z}} = 116.74 + 116.74 = \underline{\underline{233.48N}}$ ANS.

1. c) Another Method

$$\Sigma \bar{M}_A = \bar{r}_{B/A} \bar{T}_{BD} + \bar{r}_{C/A} \bar{T}_{CD} + \bar{r}_{G/A} \bar{W} = 0$$

$$= T_{BD} \times \begin{array}{ccc} (+) & (-) & (+) \\ \bar{i} & \bar{j} & \bar{k} \end{array} \begin{vmatrix} -1.5 & 0 & 2.1 \\ 0.38 & 0.76 & -0.53 \end{vmatrix} + T_{CD} \times \begin{array}{ccc} (+) & (-) & (+) \\ \bar{i} & \bar{j} & \bar{k} \end{array} \begin{vmatrix} 1.5 & 0 & 2.1 \\ -0.38 & 0.76 & -0.53 \end{vmatrix} + \begin{array}{ccc} (+) & (-) & (+) \\ \bar{i} & \bar{j} & \bar{k} \end{array} \begin{vmatrix} 0 & 0 & 1.4 \\ 0 & -500 & 0 \end{vmatrix} = 0$$

$$= -2.1 \times 0.76 T_{BD} \bar{i} - [-1.5 \times (-0.53) - 2.1 \times 0.38] T_{BD} \bar{j} - 1.5 \times 0.76 T_{BD} \bar{k}$$
$$- 2.1 \times 0.76 T_{CD} \bar{i} - [1.5 \times (-0.53) - 2.1 \times (-0.38)] T_{CD} \bar{j} + 1.5 \times 0.76 T_{CD} \bar{k}$$
$$- 1.4 \times (-500) \bar{i} = 0$$

$$= -1.60 T_{BD} \bar{i} - 0.0 T_{BD} \bar{j} - 1.14 T_{BD} \bar{k}$$
$$- 1.60 T_{CD} \bar{i} - 0.0 T_{CD} \bar{j} + 1.14 T_{CD} \bar{k} + 700 \bar{i} = 0$$

$$= -1.60 T_{BD} \bar{i} - 1.6 T_{CD} \bar{i} + 700 \bar{i} - 1.14 T_{BD} \bar{k} + 1.14 T_{CD} \bar{k} = 0$$

Now, equate coefficients of $\bar{i}$ and $\bar{j}$ to Zero:

$\bar{i}$: $-1.60\, T_{BD} - 1.6\, T_{CD} + 700 = 0$ —— (1)

$\bar{k}$: $-1.14\, T_{BD} + 1.14\, T_{CD} = 0$ —— (2)

From (2): $T_{BD} = T_{CD}$

Insert in (1):

$$-1.60 T_{BD} - 1.6 T_{BD} + 700 = 0$$

$$3.2\, T_{BD} = 700$$

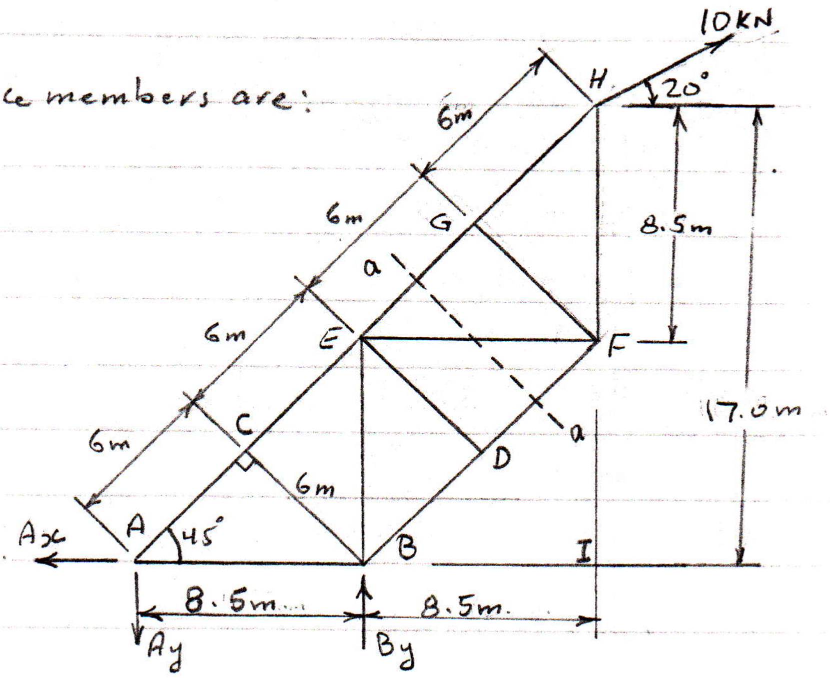
$$\therefore T_{BD} = T_{CD} = \frac{700}{3.2} = \underline{\underline{218.75 N}} = \underline{\underline{219 N}} \simeq \underline{\underline{220 N}}$$ AN

2. a)

By inspection, zero-force members are:

BC, DE and GF

b) FBD - Entire truss



$$AB = HF = \frac{6m}{\cos 45^\circ} = 8.5m$$

$BI = IF = 12 \cos 45^\circ = 8.5m.$

$\therefore HI = 17.0 m.$

$\curvearrowleft\!+ \; \Sigma M_A = 0$

$B_y \times 8.5m - 10KN \cos 20^\circ \times 17.0m + 10KN \sin 20^\circ \times 17.0m = 0$

$8.5 B_y - 159.7 + 58.1 = 0$

$8.5 B_y = 101.6, \quad \therefore B_y = \frac{101.6}{8.5} = \underline{\underline{11.95 KN \uparrow}}$ ANS.

$+\uparrow \Sigma F_y = 0$

$B_y - A_y + 10KN \sin 20^\circ = 0$

$11.95 - A_y + 3.42 = 0, \quad \therefore A_y = \underline{\underline{15.37 KN \downarrow}}$ ANS.

$\xrightarrow{+} \Sigma F_x = 0$

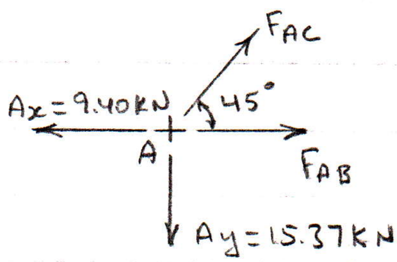
$-A_x + 10KN \cos 20^\circ = 0, \quad \therefore A_x = \underline{\underline{9.40 KN}} \leftarrow$ ANS.

c) FBD - Joint A

$+\uparrow \Sigma F_y = 0; \; F_{AC} \sin 45^\circ - A_y = 0$

$0.707 F_{AC} = A_y = 15.37$

$\therefore F_{AC} = \frac{15.37}{0.707} = \underline{\underline{21.74 KN (T)}}$ ANS.



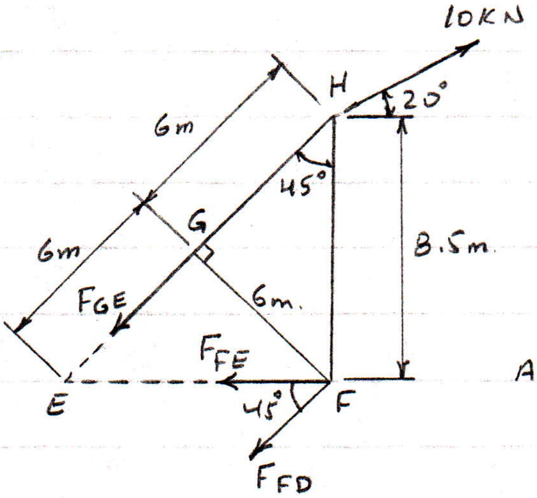
$\xrightarrow{+} \Sigma F_x = 0; \; F_{AC} \cos 45^\circ + F_{AB} - A_x = 0$

$21.74 \times 0.707 + F_{AB} - 9.40 = 0$

$\therefore F_{AB} = \underline{\underline{5.97 KN (T)}}$ ANS.

2. (Cont'd)

d) FBD - Right of Sec. a-a



$+\circlearrowleft \Sigma M_F = 0$

$F_{GE} \times 6m - 10KN \cos 20° \times 8.5m = 0$

$6 F_{GE} = 79.87$

$\therefore F_{GE} = \frac{79.87}{6} = \underline{\underline{13.3 \text{ kN (T)}}}$ ANS.

$+\circlearrowleft \Sigma M_E = 0$

$-F_{FD} \times 6m - 10KN \cos 20° \times 8.5m + 10KN \sin 20° \times 8.5m = 0$

$-6 F_{FD} = 79.87 - 29.07 = 50.8$

$\therefore F_{FD} = \frac{-50.8}{6} = \underline{\underline{8.5 \text{ kN (C)}}}$ ANS.

$\xrightarrow{+} \Sigma F_x = 0$

$-F_{FE} + F_{FD} \cos 45° - F_{GE} \cos 45° + 10KN \cos 20° = 0$

$-F_{FE} + 8.5 \cos 45° - 13.3 \cos 45° + 10 \cos 20° = 0$

$F_{FE} = +6.01 - 9.40 + 9.40$

$\therefore F_{FE} = +6.01 \text{ kN} = \underline{\underline{6.01 \text{ kN (T)}}}$ ANS.

3.

Block A $= 20\,kg \times 9.81 = 196.2\,N$

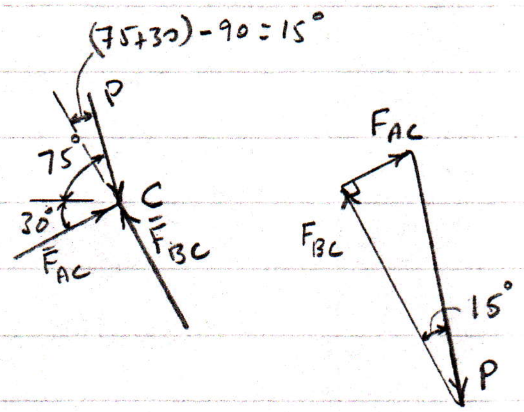
Block B $= 10\,kg \times 9.81 = 98.1\,N$

<u>FBD - Joint C</u>

Look at the Force triangle to the right:

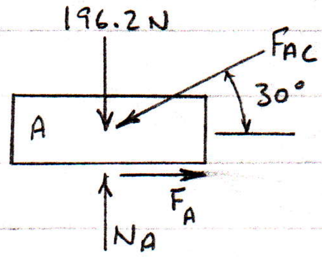
$F_{AC} = P \sin 15^\circ = 0.2588\,P$

$F_{BC} = P \cos 15^\circ = 0.9659\,P$



<u>FBD - Block A</u>

Assume that motion impending will start at block A first.



$+\uparrow \Sigma F_y = 0;\quad N_A - 196.2 - F_{AC} \sin 30^\circ = 0$

$N_A = 196.2 + 0.2588\,P \sin 30^\circ$

$N_A = 196.2 + 0.1294\,P$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad F_A - F_{AC} \cos 30^\circ = 0$

$F_A = 0.2588\,P \cos 30^\circ = 0.2241\,P$

For motion impending at A: $F_A = \mu_s N_A$

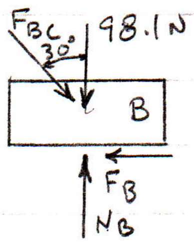
$$\text{i.e. } N_A = \frac{F_A}{\mu_s} = \frac{0.2241P}{0.2} = 1.1205\,P$$

Insert $N_A = 196.2 + 0.1294\,P$

$196.2 + 0.1294\,P = 1.1205\,P$

$$0.9911\,P = 196.2 \quad \therefore P = \frac{196.2}{0.9911} = \underline{197.96\,N}$$

<u>FBD - Block B</u> (Assume that motion impends at B)



$+\uparrow \Sigma F_y = 0;\quad N_B - 98.1 - F_{BC} \cos 30^\circ = 0$

$N_B = 98.1 + 0.9659\,P \cos 30^\circ$

$N_B = 98.1 + 0.8365\,P$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad F_{BC} \sin 30^\circ - F_B = 0$ (cont'd on next page)

3. (Cont'd)

$$F_B = F_{BC}\sin 30^\circ = 0.9659P\sin 30^\circ = 0.4830P$$

For motion impending at B:

$$F_B = \mu_s N_B$$

$$N_B = \frac{F_B}{\mu_s} = \frac{0.4830P}{0.2} = 2.415P$$

Insert $N_B = 98.1 + 0.8365P$

$$98.1 + 0.8365P = 2.415P$$

$$1.5785P = 98.1$$

$$\therefore P = \frac{98.1}{1.5785} = 62.15N$$

$\therefore$ For equilibrium to be maintained, $\underline{\underline{P = 62.15N}}$ ANS.

4. a)

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$20m = 0 + 0 + \frac{1}{2} a \times (2.5)^2$$

$$\therefore a = \frac{20 \times 2}{(2.5)^2} = \underline{\underline{6.4\ m/s^2}}$$ is the acceleration at B. ANS.

$$v = v_0 + at$$

$$v = 0 + 6.4 \times 2.5$$

$$\therefore v = \underline{\underline{16\ m/s}}$$ is the velocity at point B. ANS.

b)

- <u>Horizontal Motion</u>

$$x = x_0 + (v_0)_x t$$

$$60 = 0 + v_0 \cos 30^\circ t$$

$$v_0 t = \frac{60}{\cos 30^\circ} = 69.28 \quad \text{——— (1)}$$

- <u>Vertical Motion</u>

$$y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$$

$$0 = 0 + v_0 \sin 30^\circ t - \frac{1}{2} \times 9.81 t^2$$

$$4.9 t^2 = 0.5 v_0 t \quad \text{——— (2)}$$

Insert (1) in (2):

$$4.9 t^2 = 0.5 \times 69.28$$

$$t^2 = \frac{0.5 \times 69.28}{4.9} = 7.07$$

$$\therefore t = 2.66 s.$$

Insert in (1): $2.66 v_0 = 69.28$, $\therefore \underline{\underline{v_0 = 26.05\ m/s}}$ ANS.

$v^2 = v_0^2 + 2a(h - h_0)$ (In the vertical plane)

$$0 = (v_0)_y^2 - 2gh$$

$$0 = (26.05 \sin 30^\circ)^2 - 2 \times 9.81 h$$

$$\therefore h = \frac{(26.05 \sin 30^\circ)^2}{2 \times 9.81} = \frac{169.65}{19.62} = \underline{\underline{8.65 m}}$$ ANS.

- END -